MATHEMATICS

Instruction: You are offered the test items with one correct answer from five proposed ones.

1. Find the value of $arctg(-\dfrac{\sqrt{3}}{3})$.

A) $-\dfrac{\pi}{6}$

B) $\dfrac{\pi}{3}$

C) $\dfrac{2\pi}{3}$

D) $\dfrac{\pi}{6}$

E) $-\dfrac{\pi}{3}$

2. What is the sum of the roots of equation: $(5x-1)^2 + 4(5x-1) - 5 = 0$

A) $-\dfrac{6}{5}$

B) 4

C) 1

D) $-\dfrac{2}{5}$

E) 0

3. Given: $\begin{cases} x^2 + y^2 = 13 \\ x \cdot y = 6 \end{cases}$. Find the interval which includes all solutions of the system of equations.

A) $[-3;3]$

B) $[-10; -2]$

C) $[-5;2]$

D) $[-2;8]$

E) $[1;5]$

4. The price of book increased by 15% , and now it costs 1656tg . What is the initial price of the book?

A) 1440 tg

B) 1510 tg

C) 1220 tg

D) 1460 tg

E) 1600 tg

5. $1, \dfrac{3}{4}, \dfrac{9}{16} \ldots$ is infinite geometric sequence. Find S_n.

A) 2

B) 3

C) 5

D) 6

E) 4

6. Solve the inequality: $\cos x \ge -\dfrac{1}{\sqrt{2}}$

A) $\left(\dfrac{3\pi}{4} + 2\pi n; \dfrac{5\pi}{4} + 2\pi n\right), n \in Z$

B) $\left[-\dfrac{\pi}{4} + 2\pi n; \dfrac{\pi}{4} + 2\pi n\right], n \in Z$

C) $\left[-\dfrac{5\pi}{4} + 2\pi n; -\dfrac{3\pi}{4} + 2\pi n\right], n \in Z$

D) $\left[\dfrac{3\pi}{4} + 2\pi n; \dfrac{5\pi}{4} + 2\pi n\right], n \in Z$

E) $\left[-\dfrac{3\pi}{4} + 2\pi n; \dfrac{3\pi}{4} + 2\pi n\right], n \in Z$

7. Factorize the polynomial fraction:

$$\dfrac{6mn - 15mp - 4qn + 10qp}{3m - 2q} =$$

A) $2n - 5p$

B) $\dfrac{3+q}{3-p}$

C) $\dfrac{3p + 2n}{3m - 2q}$

D) $\dfrac{3p - 2n}{3m - 2q}$

E) $3n - p$

8. Given: $\begin{cases} 3x + 2y - 2 = 11 \\ 2y + 5x = 19 \end{cases}$. Find the value of $x + y$, if (x, y) is the solution of the system of equations.

A) 5

B) 12

C) 8

D) 18

E) 4

9. Find the equation of the tangent line of $y = 9x - x^2$ at point $x_0 = 1$.

A) $y = \dfrac{1}{2}x + 1$

B) $y = -8x + 1$

C) $y = x$

D) $y = 7x + 1$

E) $y = \dfrac{3}{4}x + 2$

10. Given a parallelogram ABCD, $\angle BCD = 120°$, the length of AD is $2\sqrt{3}$, an altitude drawn from point A to the base BC is equals 3. What is the perimeter of parallelogram?

A) $4\sqrt{3} + 6$

B) $6\sqrt{3}$

C) $8\sqrt{3}$

D) $6(\sqrt{3} + 1)$

E) $8(\sqrt{3} + 1)$

11. The expression $12x^2 - 36xy + 27y^2$ equals to:

A) $3(2x - 3y)^2$

B) $2x^2 - 3y^2$

C) $8x^2 - 12xy + 9y^2$

D) $3(8x^2 - 27y^2)$

E) $(2x - 3y)^2$

12. In a regular square pyramid basal edge equals 5 dm and height equals 6 dm. Find the apothem (slant height) of the pyramid.

A) 7 dm

B) 7.5 dm

C) 6.6 dm

D) 6.3 dm

E) 6.5 dm

13. Given the arithmetic sequence: $a_n = 3n + 5$. If $a_1 - 3, a_3 - 4, a_5$ are consecutive terms of geometric sequence, then find ratio (q) of the geometric sequence.
A) 2
B) 3
C) 6
D) 5
E) 4

14. Solve the system of inequalities:
$$\begin{cases} 24x - 8 > 0 \\ -36x + 12 > 0 \end{cases}$$
A) 0
B) $\dfrac{1}{3}$
C) No solution
D) $-\dfrac{1}{3}$
E) 3

15. Solve: $-2x + 7 < 4x - 11$
A) $(-10;8]$
B) $(3;+\infty)$
C) $(-\infty;6]$
D) $(-\infty;+\infty)$
E) $(6;12]$

16. The distance between two cities on the map with a scale 1:2500000 is 4 cm. What is a real distance between them?
A) 100 km
B) 1 km
C) 1000 km
D) 0.1 km
E) 10 km

17. Given: $\sin\alpha = \dfrac{3}{5}$ and $\alpha \in (90°;180°)$. Find the value of $\dfrac{\cos\alpha \cdot tg\alpha}{ctg\alpha}$

    A) 0

    B) 1

    C) $\dfrac{9}{20}$

    D) -1

    E) $-\dfrac{9}{20}$

18. Solve the equation: $4(2x-1)-(x-16)=2(6x-5)-5(x+2)$

    A) $(-\infty;+\infty)$

    B) $\dfrac{40}{14}$

    C) 7

    D) 0

    E) no solution

19. Solve the system of inequalities: $\begin{cases} 3^{x-2} < \dfrac{3}{\sqrt[x]{9}} \\ 6^{x+2} > 2^{x^2} \cdot 3^{x+2} \end{cases}$

    A) $(-4;0)$

    B) $(-2;+\infty)$

    C) $(0;4)$

    D) $(-2;2)$

    E) $(-1;0)\cup(1;2)$

20. Given the points: A(5;2), B(3;−3), C(2; x) and D(6;1) . If $\overrightarrow{BA}$ and $\overrightarrow{CD}$ are collinear, find the value of x.

    A) 8

    B) −9

    C) 12

    D) −6

    E) 0

Balloons

Yerlan wants to sell balloons. He has 7 different colours of balloons. Each colours have 12 balloons.

21. One balloon flied to the air. What is the probability that is a red balloon?

A) $\dfrac{1}{12}$

B) $\dfrac{2}{21}$

C) $\dfrac{1}{7}$

D) $\dfrac{2}{7}$

E) $\dfrac{1}{6}$

22. Find the colours of 24th balloon, if Yerlan put 5 colours of the balloons in this order: red,yellow, green, blue and purple.
    A) yellow
    B) red
    C) blue
    D) greeen
    E) purple

23. Find the colours of 30th balloon, if Yerlan put 7 colours of the balloons in this order: red, yellow, green, blue, purple, orange and pink.
    A) green
    B) purple
    C) red
    D) yellow
    E) blue

24. A boy wants to buy 3 balloons. In how many ways he can buy one red and two yellow balloons?
    A) 792
    B) 798
    C) 882
    D) 800
    E) 794
25. At the end of the day, there are 3 red, 7 yellow, 9 purple and 2 blue balloons are left. How many percentage of balloons have been sold?
    A) 45%
    B) 21%
    C) 25%
    D) 28 %
    E) 75%

26. Find the factors of $a^2 - a - 6$.
   A) $6 - a$
   B) $a + 1$
   C) $a - 3$
   D) $a + 6$
   E) $a - 2$
   F) $a + 3$
   G) $a + 2$
   H) $a - 6$

27. Solve the system of equations:
$$\begin{cases} x - y = 12 \\ xy = 108 \end{cases}$$
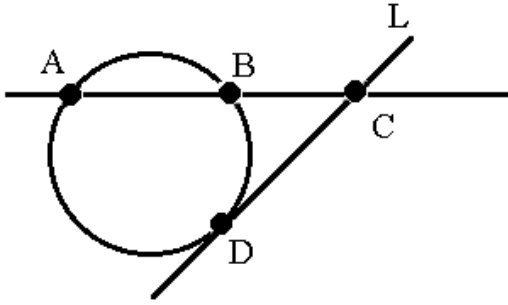   A) (-18, -6)
   B) (18, 6)
   C) (7,19)
   D) (-9,-2)
   E) (-6, -18)
   F) (19, 7)
   G) (16, 4)
   H) (4, 16)

28. Five workers can do a job for 4 days. How many days 10 workers need to do the same job?
   A) 6 days
   B) 8 days
   C) 5 days
   D) 1 day
   E) 4 days
   F) 3 days
   G) 2 days
   H) 14 days

29. In the L is the tangent line to the circle. If DC=6 cm, AC=9 cm and BC=?

A) 4 cm

B) $\sqrt{16}$ cm

C) $4\sqrt{2}$ cm

D) $\sqrt{36}$ cm

E) 10 cm

F) $\sqrt{64}$ cm

G) 6 cm

H) 8 cm

30. Find the intersection points of the graphs of $y=4x$ and $y=x^2-2x+9$.

A) $\left(\dfrac{1}{4},1\right)$

B) (3, 12)

C) $\left(\dfrac{1}{3},1\right)$

D) (4, 10)

E) (5, 20)

F) (2, 8)

G) (1, 4)

H) (14, 3)

31. Find which points lie on the graph of the function $4y + 5x^2 = 0$
   A) (0, 0)
   B) (-4, 1)
   C) (-2, -5)
   D) (-3, 2)
   E) (-3, 1)
   F) (1, 1)
   G) $\left(\dfrac{4}{5}, 1\right)$
   H) (4, 10)

32. Given recurrence formula of the sequence: $a_1=8$, $a_2=13$ and $a_{n+1} = 2a_n - a_{n-1}$.
   Which one of the followings are 3rd , 4th, and 5th terms of the sequence?
   A) 16
   B) 18
   C) 28
   D) 23
   E) 2
   F) 12
   G) 20
   H) 26

33. Evaluate: $\dfrac{4 - 3\cdot 2^{\frac{1}{2}}}{\left(2^{\frac{1}{4}} - 8^{\frac{1}{4}}\right)^2}$

   A) $\sqrt{4}$
   B) $(-1)^3$
   C) 3
   D) -1
   E) -3
   F) 1
   G) 2
   H) 1^5

34. Triangles $\triangle ABC$ and $\triangle DEF$ are similar. $|AB| = 6cm$, $|BC| = 3cm$ and perimeter of $\triangle ABC$ is 13 cm. If $|DE| = 18cm$ and $|EF| = 9cm$. Find the length of $|DF|$.
   A) 0.12 m
   B) 12 cm
   C) 1 cm
   D) 0.01 m
   E) 9.5 cm
   F) 14 cm
   G) 8 cm
   H) 10 cm

35. The object moves with $s(t) = \dfrac{1}{2}t^2 - \dfrac{1}{2}t$ (s(t) - meter, t – time). The velocity of the object reaches the highest value at exact time in a range
   [1s; 10s] (time, s). What time is that and what is the highest velocity?
   A) $t = 8s$
   B) $t = 7s$
   C) $t = 10s$
   D) $\upsilon = 10,5m/s$
   E) $t = 7,5s$
   F) $\upsilon = 14m/s$
   G) $\upsilon = 13,5m/s$
   H) $\upsilon = 9,5m/s$